SHU

2010
MATHEMATICS - II (Optional)

200045

Standard : Degree | Total Marks : 200

Nature : Conventional | Duration : 3 Hours

Note :

(i) *Answers must be written in **English** only.*

(ii) *Question **No. 1** is **Compulsory**. Of the remaining questions, attempt **any four** selecting one question from **each section**.*

(iii) *Figures to the **RIGHT** indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table / Code / Reference book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing / presentation.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

1. Answer ***any four*** of the following :

(a) Prove that, if H is a p-Sylow subgroup of a group G and $x \in G$, then $x^{-1}Hx$ is also a p-Sylow subgroup of G. **10**

(b) Find the residue of the function. **10**

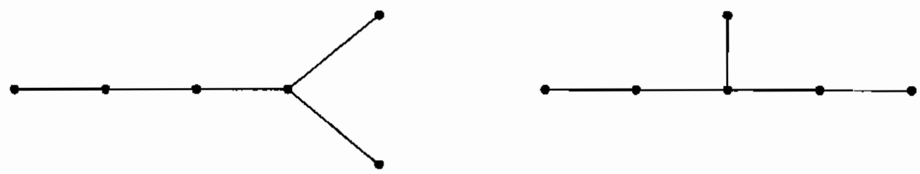
$$\frac{1}{\left(z^2+1\right)^3} \text{ at } z=i.$$

(c) Find the general solution of $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$. **10**

P.T.O.

Marks

(d) Are the following two graphs isomorphic ? Justify your answer. **10**



(e) Prove that if f is continuous function on [a, b] and if f (a) and f (b) are of opposite signs, then there exists a point c ϵ (a, b) such that f (c) = 0. **10**

SECTION – A

2. Answer the following sub-questions :

(a) Prove that if p is prime number and a is any integer then $a^p \equiv a$ mod p. **15**

(b) Prove that every finite integral domain is field. **15**

(c) Prove that, for every prime number p and every positive integer m there is a unique field having p^m elements. **10**

3. Answer the following sub-questions :

(a) Prove that the order of every element of a finite group is finite and is less than or equal to the order of the group. **15**

(b) Prove that the ring of integers is a principal deal ring. **15**

(c) Let R be the field of real numbers and Q the field of rational numbers. In R, $\sqrt{2}$ and $\sqrt{3}$ are both algebraic over Q. Exhibit a polynomial of degree 4 over Q satisfied by $\sqrt{2} + \sqrt{3}$. **10**

Marks

SECTION – B

4. Answer the following sub-questions :

(a) Prove that every bounded sequence of real numbers has a convergent sub-sequence. **15**

(b) Prove that the series $\sum(-1)^n\left[\sqrt{n^2+1}-n\right]$ is conditionally convergent. **15**

(c) Prove that a necessary condition that a function **10**

$$f(z)=U(x, y)+i\,V(x, y)$$

be analytic at a point $z=x+iy$ of its domain D is that at a point (x, y) the first order partial derivatives of U and V with respect to x and y exist and satisfy the Cauchy-Riemann equations

$$U_x=V_y \text{ and } U_y=-V_x.$$

5. Answer the following sub-questions :

(a) Applying Cauchy's criterion of convergence, prove that the sequence $\{S_n\}$ defined by **15**

$$S_n=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\text{........}+\frac{(-1)^{n-1}}{n} \text{ is convergent.}$$

(b) Prove that, if the series $\sum an$ and $\sum bn$ are convergent, then $\sum\sqrt{anbn}$ is also convergent. Show by example that $\sum\sqrt{anbn}$ may converge, even if $\sum an$ and $\sum bn$ are divergent. **15**

(c) Evaluate $\int_c \frac{z^3}{z-2i}\,dz$, where c is circle $|z-2|<5$, by using Cauchy's integral formula. **10**

Marks

(d) Use simplex method to maximize 10

$$z = -x_1 + 3x_2 - 2x_3$$

subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

-oOo-

Marks

SECTION – C

6. Answer the following sub-questions :

(a) Determine the general solution of the differential equation : **15**

$$\frac{d^2 y}{dx^2} - 4y = e^x + \sin 2x$$

(b) If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$. **10**

(c) Find integral surface of the equation : **15**

$$(x - y)\, y^2 \frac{dz}{dx} + (y - x)\, x^2 \frac{dz}{dy} = x^2 + y^2$$

through the curve $xz = a^2$, $y = 0$.

7. Answer the following sub-questions :

(a) Determine general solution of the differential equation **15**

$$\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$$

(b) If $\theta = t^n e^{-r^2/4t}$, find values of n for which $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$. **10**

(c) Find complete solution of $(p^2 + q^2)y = qz$, by Charpit's method. **15**

P.T.O.

Marks

SECTION – D

8. Answer the following sub-questions :

(a) Perform five iterations of the bisection method to find smallest positive root of the equation $x^3 - 5x + 1 = 0$, given that the root lies between 0 and 1. **10**

(b) Use Gauss-Seidal iteration method to solve : **10**

$$2x_1 - x_2 = 7.$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

by performing three iterations.

(c) Using Trapezoidal rule find $\int_0^2 \left(1 + 4x^2\right) dx$ by dividing [0, 2] into 4 equal subintervals. Find also error in your answer. **10**

(d) Use simplex method to maximize $z = 5x_1 + 3x_2$ subject to the constraints $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1, x_2 \geq 0$. **10**

9. Answer the following sub-questions :

(a) Use Newton Raphson method to find a real root of the equation $x^3 - x - 4 = 0$, which lies between 1 and 2 correct upto three places of decimals. **10**

(b) Using Newton's forward difference interpolation formula, estimate $f(8)$, from the data : **10**

x	5	10	15	20
$f(x)$	50	70	100	145

(c) Find an approximate value of $\log_e 2$ by evaluating $\int_1^2 \frac{1}{x} dx$ using Simpson's one–third rule taking five ordinates. **10**

P.T.O.