2009

STATISTICS - I (Optional)

100071

Standard : Degree *Total Marks : 200*

Nature : Conventional *Duration : 3 Hours*

Note :

(i) *Answers must be written in* **English** *only.*

(ii) *Question* **No. 1** *is* **Compulsory.** *Of the remaining questions, attempt* **any four** *selecting one question from* **each section.**

(iii) *Figures to the* **RIGHT** *indicate marks of the respective question.*

(iv) *Though use of log table, Non-Programmable calculator is permitted, but any other Table/Code/ Reference book are not permitted.*

(v) *Make suitable assumptions, wherever be necessary and state the same.*

(vi) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vii) *Credit will be given for orderly, concise and effective writing.*

(viii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

1. Answer *any four* of the following (**10** marks each) :

(a) Show that for normal distribution all odd - ordered central moments are zero. **10**

(b) Let X_1, X_2 be a random sample of size 2 from the distribution with p.d.f. **10**

$f(x,\theta) = \theta x^{\theta - 1}$; $0<x<1$; $\theta>0$

$= 0$; otherwise

To test $H_0:\theta = 1$ Vs $H_1:\theta = 2$, the following critical region is used $\left\{(x_1, x_2)/x_1 \cdot x_2 \geq \frac{3}{4}\right\}$ obtain (i) size of the test, (ii) power of the test

(c) In usual notations, prove that **10**

$$R^2_{1.23} = \frac{r^2_{12} + r^2_{13} - 2\, r_{12}\, r_{13}\, r_{23}}{1 - r^2_{23}}$$

P.T.O.

Marks

(d) In simple random sampling without replacement, in usual notations, prove that 10

(i) $V(\bar{y}) = \frac{1-f}{n} S^2$

(ii) $E(8^2) = S^2$

(e) Describe sign test based on single sample for testing the value of population median, stating clearly the underlying assumptions. 10

SECTION - A

2. Answer the following sub - questions :

(a) (i) Show that, under some condition (s), binomial distribution tends to poisson distribution. 5

(ii) State central limit theorem for iid random variables. Hence, obtain sampling distribution of sample proportion. 5

(b) The joint p.d.f. of (X,Y) is given by 10

$f(x,y) = 1$; $0<x<1$; $0<y<1$

$= 0$; otherwise

obtain (i) $P(X>2Y)$, (ii) $P\left(X > Y / Y > \frac{1}{2}\right)$ (iii) $P\left(X^2+Y^2 < \frac{1}{4}\right)$

(c) If $P_k = P(x=k)$ denotes the probability that there are K failures preceding the first success, obtain p.g.f. of P_k. Hence, obtain its mean and variance. 10

(d) Let $\{x_n\}$ be any sequence of random variables Let $Y_n = \Sigma xi/n$. Prove that the necessary and sufficient condition for the sequence $\{x_n\}$ to satisfy WLLN is 10

$$E\left\{\frac{Y_n^2}{1+Y_n^2}\right\} \to 0 \text{ as } n \to \infty$$

3. Answer the following sub - questions :

(a) (i) An elevetor starts with 5 passengers and stops on all 7 floors. What is the probability that each passenger leaves on different floor ? 5

(ii) If $f(x) = 504\, x^5(1-x)^3$; $0<x<1$ then using chebyshev's inequality obtain lower bound on $P[0.4<x<0.6]$. 5

(b) If $f(x,y) = \frac{1}{25}\left(\frac{20-x}{x}\right)$; $10<x<20$, $\frac{x}{2}<y<x$ 10

$= 0$; otherwise

obtain conditional p.d.f. of X given $Y=y$ and conditional p.d.f. of Y given $X=x$.

(c) Obtain m.g.f. of exponential distribution with parameter θ. Hence, obtain an expression for r^{th} ordered raw moment of the distribution. 10

Marks

(d) When do you say that the sequence $\{x_n\}$ of random variables obeys WLLN ? If $\{x_n\}$ is a sequence of pairwise uncorrelated random variables with $E(x_i) = \mu i$ and $V(x_i)\ \sigma_i^2$ and if $\Sigma\sigma_i^2 \to \infty$ as $n \to \infty$ then prove that $\Sigma\frac{(x_i - \mu_i)}{\Sigma\sigma_i^2} \xrightarrow{p} 0$ as $n \to \infty$. **10**

SECTION - B

4. Answer the following sub - questions :

(a) State and prove Cramer - Rao in equality. **10**

(b) (i) Let a random sample of 5 observations 8.1, 0.2, 1.6, 5.2, 2.1 be taken from a population with p.d.f. **5**

$$f(x,\alpha,\beta) = \frac{1}{\beta - \alpha};\ \alpha < x < \beta$$

Estimate α and β by method of moments

(ii) State Neymann's factorisation theorem. Obtain sufficient estimator of θ based on random sample of size n drawn from the distribution with p.d.f. **5**

$$f(x,\theta) = \frac{1}{\theta}\, e^{-x/\theta} \quad x>0,\ \theta>0$$
$$= 0 \quad ;\ \text{otherwise}$$

(c) Obtain Uniformly most powerful test of size α for testing $H_0 : \theta = \theta_0$ Vs. $H_1 : \theta > \theta_0$ based on a random sample of size n drawn from the population with density $N(0,\theta)$. Also, obtain power function of the test. **10**

(d) Describe Kolmogorov - smirnov test for goodness of fit, stating clearly the underlying assumptions. **10**

5. Answer the following sub-questions :

(a) Define Minimum Variance Unbiased Estimator (MVUE) Prove that MVUE, if exists, is unique. **10**

(b) (i) If $f(x, \theta) = e^{-(x-\theta)}$; $x \geq \theta$, $\theta > 0$, obtain MLE of θ based on sample of size n. Is this estimator unbiased for θ ? Justify. **5**

(ii) Let $X_1, X_2, \ldots, Xn$ be a random sample from a distribution with p.d.f. $f(x, \theta) = \theta\,(1-\theta)^{x-1}$; $x = 1, 2, 3, \ldots$, $\theta > 0$ show that variance of sample mean attains CRLB. **5**

(c) A random sample of size 1 is taken from the distribution that has p.d.f. **10**

$$f(x,\theta) = \frac{2}{\theta^2}(\theta - x)\ ;\ o < x < \theta$$
$$= \quad o \quad ;\ \text{elsewhere}$$

Obtain Likelihood Ratio Test of size α to test
$Ho : \theta = \theta_0$ Vs $H_1 : \theta \neq \theta_0$

(d) Develop SPRT of strength (α, β) for testing $Ho : \theta = \theta_0$ against $H1 : \theta = \theta_1$ $(\theta_1 < \theta_0)$ based on a random sample drawn from a population following exponential distribution with parameter θ. **10**

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SECTION - C

6. Answer the following sub-questions :

(a) For bivariate linear regression of Y on X, obtain the expressions for its constants using principle of least squares. **10**

(b) The following table gives the weights (X_1) in pounds, height (X_2) in inches and age (X_3) in years of 12 boys. **10**

x_1	64	71	53	67	55	58	77	57	56	51	76	78
x_2	57	59	49	62	51	50	55	48	52	42	61	57
x_3	8	10	6	11	8	7	10	9	10	6	12	9

Obtain the least square regression of X_1 on X_2 and X_3 and estimate the weight of a boy who is 9 years old and 54 inches. tall.

(c) If the joint p.d.f of (X, Y) is bivariate normal with parameters (0, 0, 1, 1, ρ), obtain. **10**

(i) marginal distribution of X,

(ii) conditional distribution of Y given $X=x$

(iii) distribution of $X+Y$.

(d) For a 2×2 contingency table, describe odds ratio as a measure of association. State its any four properties. **10**

7. Answer the following sub-questions :

(a) Describe scatter diagram as a method of assessing correlation between two variables X and Y. State its limitation (s). In view of this limitation (s), discuss coefficient of correlation. **10**

(b) (i) "Correlation is not necessarily a Causation". Explain with suitable illustrations. **5**

(ii) The following data give total dissolved solids (X_1), calcium (X_2) and electrical conductivity (X_3) in 10 randomly selected water samples collected from a river in India : **5**

x_1	210	230	260	165	195	290	156	222	237	245
x_2	36.0	44.8	43.6	32.0	37.6	52.0	32.0	27.2	31.0	38.4
x_3	257	322	287	189	362	336	304	300	320	312

Obtain multiple correlation coefficient $R_{3.12}$ and test its significance.

(c) If (X, Y) follow a trinomial distribution, derive its joint m.g.f. Hence, or otherwise obtain. **10**

(i) Coefficient of correlation between X and Y,

(ii) p.d.f. of $X+Y$.

(d) Give any two examples of categorical data. How would you test independence of two attributes ? Describe any two measures of association between two attributes. **10**

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SECTION - D

8. Answer the following sub-questions :

(a) (i) Describe the procedure of stratified random sampling. What is meant by optimum allocation in stratified sampling ? **5**

(ii) In linear systematic sampling, assuming $N = nk$ show that $E\left[\bar{Y}sys\right] = \bar{Y}$. **5**

Also show that $V\left[\bar{Y}sys\right] = \frac{N-1}{N}S^2 - K\frac{(n-1)}{N}.S^2wsys$.

(b) In two stage sampling, by making suitable assumptions, prove that $V\left(\bar{\bar{Y}}\right) = \frac{1-f_1}{n}S_1^2 + \frac{1-f_2}{mn}S_2^2$. **10**

(c) What are the principal steps in conducting a sample survey ? **10**

(d) Write a brief note on 'Indian Official Statistics'. **10**

9. Answer the following sub-questions :

(a) (i) In stratified random sampling, show that in general sample mean does not estimate population mean unbiasedly. Obtain the condition under which sample mean estimates population mean unbiasedly. Name this condition. Obtain the Variance of sample mean in such case. **5**

(ii) In ratio method of estimating population mean, show, in usual notations, that $E\left[\hat{R}\right] \neq R$. Obtain an expression for exact amount of bias in $\hat{R}$. In which situation this bias is negligible ? **5**

(b) Explain the problem in estimating variance of sample mean in systematic sampling. Give any one procedure to deal with the problem. **10**

(c) What are the requisites of a good questionnaire ? Illustrate with suitable situations (s). **10**

(d) Write a brief note on National sample survey organisation (NSSO) in India. **10**

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