QTH

2009

100073

MATHEMATICS - II (Optional)

Standard : Degree | **Total Marks : 200**

Nature : Conventional | **Duration : 3 Hours**

Note :

(i) *Answers must be written in* ***English*** *only.*

(ii) *Question* ***No. 1*** *is* ***Compulsory.*** *Of the remaining questions, attempt* ***any four*** *selecting one question from* ***each section.***

(iii) *Figures to the* ***RIGHT*** *indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference Book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing/presentation.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

Marks

1. Answer **any four** of the following :

(a) Prove that any permutation of n elements is a product of disjoint cycles. Express the permutation (4 2 1 5) (3 4 2 6) (5 6 7 1) as a product of disjoint cycles. **10**

(b) State Cauchy's residue theorem. Use this theorem to evaluate the integral. **10**

$$\int_c \frac{5z-2}{z\,(z-1)}\,dz$$

where c is the circle $|z| = 2$, described in counter clockwise.

(c) Solve the one-dimensional heat flow equation : **10**

$$\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$$

subject to boundary conditions :

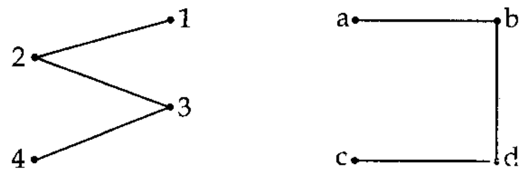
$u\,(0, t) = 0, \quad t \geqslant 0$

$u\,(80, t) = 0, \quad t \geqslant 0$ and the initial condition $u\,(x, 0) = 100 \sin \frac{\pi}{80} x$, $0 \leq x \leq 80$

P.T.O.

Marks

(d) Show that the following two graphs are isomorphic. 10



(e) Prove that the union of two closed sets in $\mathbb{R}$ is a closed set. Is the set $[2, 3)$ closed in $\mathbb{R}$? Justify. 10

SECTION - A

2. Answer the following sub-questions :

(a) Prove that a non empty subset H of a group G is a subgroup of G if and only if for all $a, b, \in H$, $ab^{-1} \in H$. Hence show that the set of even integers is a subgroup of the additive group $\mathbb{Z}$ of integers. 15

(b) Prove that in a non zero commutative ring R with unity, an ideal M is maximal if and only if $\frac{R}{M}$ is a field. Use this to show that the set $3\mathbb{Z}$ is a maximal ideal of the ring of integers $\mathbb{Z}$. 15

(c) Prove that the number of elements in a finite field F is p^n for some prime p and positive number n. Illustrate this result by an example. 10

3. Answer the following sub-questions :

(a) State and prove the first isomorphism theorem of groups. 15

(b) Prove that every principal ideal domain is a unique factorization domain. Show by an example that the converse is not true. 15

(c) Prove that if $F \subseteq E \subseteq K$ are fields such that $[K : E]$ and $[E : F]$ are finite then K is a finite extension of F. 10

SECTION - B

4. Answer the following sub-questions :

(a) Prove that every Cauchy's sequence of real numbers is convergent. What about the converse ? Justify your answer. 15

(b) Define and illustrate the concept of absolutely convergent series of real numbers. Prove that if Σa_n is absolutely convergent then Σa_n is convergent. 15

(c) State the Cauchy-Riemann equations for determining the analyticity of the functions. Use these equations to show that the complex function $f(z) = z^2$ is analytic at every point of the complex plane. 10

Marks

5. Answer the following sub-questions :

(a) Prove that a continuous function f on a closed interval [a, b] is bounded. Also, show that a constant function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2$ for all $x \leftarrow \mathbb{R}$ is continuous. 15

(b) Prove that a sequence $\{f_n\}$ of real valued functions on a set E is uniformly convergent if and only if given $\epsilon > 0$ there exists number N such that for all m, n $\geqslant$ N and for all $x \in E$

$| f_m(x) - f_n(x) | < \epsilon$. 15

(c) Find the Laurent's series of the function $f(z) = \dfrac{1}{z^2(1-z)}$ in the annulus $0 < |z| < 1$. 10

SECTION - C

6. Answer the following sub-questions :

(a) Determine the general solution of the differential equation : 15

$$\frac{d^3Y}{dx^3} + \frac{d^2Y}{dx^2} - \frac{dY}{dx} - Y = \sin(2x-3).$$

(b) If $z = z(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that : 10

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2}$$

(c) Find the surface which intersects the surfaces of the family $z(x+y) = c(3z+1)$ orthogonally and passes through the curve $(x+y) = 0$, $z = 0$. 15

7. Answer the following sub-questions :

(a) Obtain general solution of the differential equation : 15

$$\frac{d^2Y}{dx^2} - 4\frac{dY}{dx} + 3Y = \sin 3x \cos 2x.$$

(b) The radius of base of the right circular cone is 1 metre and its height is 5 metres. However, these were respectively measured as 1.03 metres and 4.9 metres. Evaluate : 10

(i) absolute error

(ii) relative error and

(iii) percentage error

in calculating the volume of the right circular cone.

(c) Find the integral surface of the linear partial differential equation : 15

$$x(y^2+z)\frac{\partial z}{\partial x} - y(x^2+z)\frac{\partial z}{\partial y} = (x^2-y^2)z$$

P.T.O.

Marks

SECTION - D

8. Answer the following sub-questions :

(a) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one real root between 1.4 and 1.5. Use false position method to find the root of this equation correct to four places of decimal. **10**

(b) Use Gauss-Seidel iteration method to solve the following system of linear simultaneous equations : **10**

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

(c) The speed v metres per second of a car, t seconds after it starts is given in the following table : **10**

t :	0	12	24	36	48	60	72	84	96	108	120
v :	0	3.60	10.08	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Use Simpson's one-third rule to find the distance travelled by the car in two minutes.

(d) Use simplex method to solve the following linear programming problem : **10**

Maximize $z = 2x_1 + 5x_2$

subject to conditions.

$$x_1 + 4x_2 \leq 24$$
$$3x_1 + x_2 \leq 21$$
$$x_1 + x_2 \leq 9$$

and $x_1, x_2 \geqslant 0$.

9. Answer the following sub-questions :

(a) The root of the equation $x^4 + x^3 - 7x^2 - x + 5 = 0$ lies between 2 and 3. Use Newton-Raphson method to determine it correct to three decimal places. **10**

(b) Apply Gauss-Seidel Method, starting with initial approximations $x = 0.3$, $y = 0.8$ and $z = 0.3$, to solve the following system of equations : **10**

$$2x - y + 2z = 3$$
$$x + 3y + 3z = -1$$
$$x + 2y + 5z = 1$$

(c) Following table gives the values of $\log x$ for $4 \leq x \leq 5-2$. **10**

x :	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$\log x$:	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

Evaluate $\int_4^{5.2} \log x \, dx$ by using Simpson's one-third rule.

(d) Use simplex method to solve the following linear programming problem : **10**

Maximize $z = x_1 + x_2 + 3x_3$

subject to conditions :

$$3x_1 + 2x_2 + x_3 \leq 3$$
$$2x_1 + x_2 + 2x_3 \leq 2$$

and $x_1, x_2, x_3 \geqslant 0$

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