QTG

100071

2009

MATHEMATICS - I (Optional)

Standard : Degree | *Total Marks : 200*

Nature : Conventional | *Duration : 3 Hours*

Note :

(i) *Answers must be written in* **English.**

(ii) *Question* **No. 1** *is* **Compulsory.** *Of the remaining questions, attempt* **any four** *selecting one question from* **each section.**

(iii) *Figures to the* **RIGHT** *indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing / presentation.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

1. Answer **any four** of the following : **Marks**

(a) Determine the eigen values and eigen vectors of the following matrix A; where. **10**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

(b) Let $f(x) = x^2, \forall x \in [0, 1]$. Show that f is Reimann integrable on $[0, 1]$. **10**

(c) Show that $\nabla^2 (r^n \bar{r}) = n(n+3)r^{n-2}\bar{r}$, **10**

where $\bar{r} = xi + yj + zk$ and $r = |\bar{r}|$.

(d) State and prove D'Alembert's principle. **10**

(e) Define asymptote of a curve. Determine the asymptotes of the cubic curve, **10**

$2x^3 - x^2y + 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$

P.T.O.

Marks

SECTION - A

2. (a) State and prove the necessary and sufficient condition for subspace of a vector space. **10**

   (b) Define similar matrices. Show that similar matrices have the same determinants and same eigen values. **10**

   (c) Define Hermitian and skew-Hermitian matrices. Illustrate Hermitian and skew-Hermitian matrices by proper examples. **10**

   (d) Define the quadratic form. Determine whether the quadratic form $q(x_1, x_2, x_3) = x_1^2 + x_2^2 - x_3^2 + 2x_1x_2$ is positive definite ? or negative definite ? or indefinite ? **10**

3. (a) Show that $B = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$ is a basis for R^3. Find an orthonormal basis. **10**

   (b) Solve the following system of linear equations by Gaussian elimination method. **10**

$$x_1 + 2x_2 - x_3 + 2x_4 = 1$$
$$3x_1 + x_3 + 4x_4 = -1$$
$$x_1 - x_2 + x_3 + x_4 = -1$$

   (c) Show that any square matrix can be represented uniquely as a sum of a symmetric and a skew symmetric matrix. **10**

   (d) Define Hermitian form. Prove that a Hermitian form X*HX is positive definite if and only if all the principal minors of H are positive. **10**

SECTION - B

4. (a) State and prove Taylor's theorem in the Lagrange's form of remainder. Hence expand $f(x) = 2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$. **15**

   (b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x-y}\right)$, $x \neq y$, show that **15**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\, \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$$

   (c) Find the area between the curves $y^2 = 4x$ and $2x - 3y + 4 = 0$. **10**

Marks

5. (a) State Langrange's conditions for maximum and minimum values of a function of two variables. Hence show that minimum value of $4 = f(x, y) = xy + (a^3/x) + (a^3/y)$ is $3a^2$. 15

(b) Define Jacobian and find $\frac{\partial (x, y)}{\partial (r, \theta)}$ and $\frac{\partial (r, \theta)}{\partial (x, y)}$, if $x = r\cos\theta$, $y = r\sin\theta$. 15

(c) Show that the volume of solid generated by revolving about x-axis the region bounded by $y = \log x$, $y = 0$ and $x = 2$ is $2\pi(1-\log 2)^2$. 10

SECTION - C

6. (a) Show that the general equation of second degree in x, y represents a conic. 10

(b) Find equation of sphere through the points (0, 0, 0), $(-a, b, c)$, $(a, -b, c)$ and $(a, b, -c)$ and determine its radius. 15

(c) Prove that $\nabla \cdot (\bar{f} x \bar{g}) = (\nabla x \bar{f}) \cdot \bar{g} - (\nabla x \bar{g}) \cdot \bar{f}$. 15

7. (a) Derive transformation equations from cartesian to spherical polar coordinates and vice versa in 3 dimensions. 10

(b) Find shortest distance between two lines 15

$$\frac{x-2}{2} = \frac{y-1}{4} = \frac{z+1}{-1} \text{ and } \frac{x+31}{3} = \frac{y-6}{2} = \frac{z-3}{6}.$$

(c) Define grad ϕ for scalar valued point function ϕ and show that 15

$$\text{grad } \phi = \frac{\bar{r} - (\bar{k} \cdot \bar{r}) R}{\left(\bar{r} - (\bar{k} \cdot \bar{r}) \bar{k}\right) \cdot \left(\bar{r} - (\bar{k} \cdot \bar{r}) \bar{k}\right) K} \text{ where } \phi(x, y) \text{ z } \log \sqrt{x^2+y^2}.$$

SECTION - D

8. (a) Define simple Harmonic motion. 15

The speed v of a point p which moves in a line is given by $v^2 = a + 2bx - cx^2$, where x is the distance of the point p from a fixed point on the path and **a, b, c** are constants. Show that the motion is simple harmonic, if **c** is positive and determine the period in terms of **a, b** and **c**.

P.T.O.

Marks

(b) Derive cartesian equation of common catenary. Show that the radius of curvature at any point p of the catenary varies as the square of the distance of p from directrix. 15

(c) Show that the moment of inertia of circular disc of radius a about an axis through centre and perpendicular to the disc is $M\frac{a^2}{2}$ where M is the mass of the disc. 10

9. (a) Show that in a central force field, the areal velocity is conserved. Hence deduce the Kepler's second law of planetary motion. 15

(b) State and prove principal of virtual work for a rigid body. 15

(c) A uniform sphere rolls down an inclined plane rough enough to prevent any sliding. Find the motion. 10

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