

2007  
STATISTICS - I (Optional)

100050

*Standard : Degree**Total Marks : 200**Nature : Conventional**Duration : 3 Hours***Note :**

- (i) *Answers must be written in English only.*
- (ii) *Question No. 1 is Compulsory. Of the remaining questions, attempt any four selecting one question from each section.*
- (iii) *Figures to the RIGHT indicate marks of the respective question.*
- (iv) *Though use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference book are not permitted.*
- (v) *Make suitable assumptions, wherever be necessary and state the same.*
- (vi) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*
- (vii) *Credit will be given for orderly, concise and effective writing.*
- (viii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

1. Answer *any four* of the following :

- (a) The number of accidents in a year attributed to taxi drivers in a city follows poisson distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with : 10
  - (i) no accident in a year
  - (ii) more than 3 accidents in a year

(Given  $e^{-3} = 0.0498$ )
- (b) For a random sampling from normal distribution  $N(\mu, \sigma^2)$ . Find the maximum likelihood estimator for : 10
  - (i)  $\mu$  when  $\sigma^2$  is known
  - (ii)  $\sigma^2$  when  $\mu$  is known

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- Marks**  
**10**
- (c) Calculate :  
 (i)  $r_{23.1}$   
 (ii)  $r_{13.2}$   
 (iii)  $r_{12.3}$   
 if  $r_{12} = 0.70$ ,  $r_{13} = 0.61$ ,  $r_{23} = 0.40$
- (d) A simple random sample of size 65 was drawn in the process of estimating the mean annual income of 950 families of a certain township. The mean and standard deviation of the sample were found to be Rs. 4730 and Rs. 765 respectively. Find 95% confidence interval for the population mean.  
 (Given  $Z = 1.96$  at 95% confidence level) **10**
- (e) The following is an arrangement of 25 men (M) and 15 women (W) lined upto purchase tickets for a premier picture show :  
 M, WW, MMM, W, MM, W, M, W, M, WWW, MMM, W, MM, WWW, MMMMMM, WWW, MMMMMM. **10**  
 Test for randomness at the 5 percent level of significance.  
 (Given  $Z = 1.96$  at 5% level of significance).

### SECTION - A

2. *Answer the following sub-questions :* **5**
- (a) (i) A random variable X has the probability function given below :  
 $f(x) = K$  if  $x = 0$   
 $= 2K$  if  $x = 1$   
 $= 3K$  if  $x = 2$   
 $= 0$  otherwise  
 (1) Determine the value of K  
 (2) Evaluate  $P(X < 2)$   
 (3) Obtain the distribution function
- (ii) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. **5**
- (b) A two dimensional variable (X, Y) has the following probability density function : **10**  
 $f(x, y) = 2 - x - y, 0 \leq x \leq 1, 0 \leq y \leq 1 = 0$  otherwise.  
 Find (i) Marginal probability density functions of X and Y.  
 (ii) Variance (X) and variance (Y)
- (c) If X denotes the outcome when a fair die is thrown. Find the moment generating function (MGF) of X and hence find the mean and variance. **10**
- (d) State and prove Weak law of large numbers. **10**

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3. Answer the following sub-questions :

- (a) (i) A continuous variable X has the following distribution function : 5

$$f(x) = 0 \text{ if } x \leq 0$$

$$= x \text{ if } 0 \leq x \leq 1$$

$$= 1 \text{ if } 1 \leq x$$

Find the probability density function.

- (ii) Let  $\{X_k\}$  be mutually independent and identically distributed random variable with mean  $\mu$  and finite variance. If  $S_n = X_1 + X_2 + \dots + X_n$  prove that law of large numbers does not hold for the sequence  $S_n$ . 5

- (b) A random variable X has the following probability mass function. (p.m.f.) 10

X	-2	3	1
P(X=x)	1/3	1/2	1/6

- Find : (i) First four raw moments.  
(ii) First four central moments.

- (c) A random variable X has the following probability density function 10

$$f(x) = K.e^{-Kx} \quad K > 0, x > 0$$

$$= 0 \text{ otherwise}$$

Find the moment generating function and hence find the mean and variance.

- (d) State and prove chebychev's inequality. 10

**SECTION - B**

4. (a) Obtain the minimum variance unbiased estimator (MVUE) for  $\mu$  in the normal population  $N(\mu, \sigma^2)$  where  $\sigma^2$  is known. 10

- (b) (i) Let  $x_1, x_2, \dots, x_n$  be a random sample from  $N(\mu, \sigma^2)$  population. Find sufficient estimator for  $\mu$  and  $\sigma^2$ . 5

- (ii) Estimate  $\alpha$  and  $\beta$  by the method of moments for the probability density function. 5

$$f(x, \alpha, \beta) = \frac{\beta^\alpha}{\sqrt{(\alpha)}} x^{\alpha-1} e^{-\beta x}, 0 \leq x \leq \infty$$

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## SECTION - D

8. (a) (i) Write short note on simple random sampling. 5  
(ii) Write short note on cluster sampling. 5
- (b) Write short note on systematic sampling. State its merits and demerits. 10
- (c) What are non-sampling errors ? What are the sources of non-sampling errors ? 10
- (d) Discuss briefly National Sample Surveys. 10
9. (a) (i) What do you mean by Stratified Sampling ? 5  
(ii) What do you mean by Two-Stage Sampling ? 5
- (b) State and explain the principal steps in the planning and executions of the Sample Survey. 10
- (c) It is desired that an accuracy of 2% is obtained with confidence limits of 95%. Out of 200 observations success was obtained in 30. Find the required size of the sample (Given  $Z=1.96$  at 95% confidence level). 10
- (d) Discuss briefly population statistics. 10

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- (c) 200 digits were selected at random from a set of tables. The frequencies of the digits were 10

Digit	0	1	2	3	4	5	6	7	8	9
$f$	18	19	23	21	16	25	22	20	21	15

use the chisquare ( $X^2$ ) test to assess the correctness of the hypothesis that the digits were distributed in equal numbers in the table from which these were chosen.

(Given  $X^2$  at 5% level for 9 d.f. = 16.919.)

- (d) Let  $X$  be  $N(\theta, 100)$ . To find the sequential probability ratio test for testing  $H_0: \theta = 75$  against  $H_1: \theta = 78$  such that each  $\alpha$  and  $\beta$  approximately equal to 0.10. 10

5. (a) State and prove Rao-Blackwell theorem. 10

- (b) (i) Let  $x_1, x_2, \dots, x_n$  be a random sample from a normal population  $N(\mu, 1)$  5

show that  $t = \frac{1}{\mu} \sum_{i=1}^n x_i^2$  is an unbiased estimator of  $1 + \mu^2$ .

- (ii) Given the frequency distribution : 5

$$f(x, \theta) = \frac{1}{\theta}, \quad 0 \leq x \leq \infty$$

$$= 0 \quad \text{otherwise}$$

and that you are testing the null hypothesis  $H_0: \theta = 1$  against  $H_1: \theta = 2$  by means of a single observed values of  $x$  what would be the sizes of type I and type II errors, if  $0.5 \leq x$ . Also obtain the power function of the test.

- (c) Two random samples are drawn from two normal populations and the following results were obtained. 10

Sample I	16	17	18	19	20	21	22	24	26	27		
Sample II	19	22	23	25	26	28	29	30	31	32	35	36

Obtain estimates of the variances of population and test whether the two populations have the same variance.

(Given  $F_T$  at 5% level for (11, 9) d.f. = 3.10)

- (d) State and explain sign test. Use the sign test to solve the following problem. 10

If there is a difference between the number of days until collection of an account receivable before and after a new policy. Use 0.05 significance level.

Before	30	28	34	35	40	42	33	38	34	45	28	27	25	41	36
After	32	29	33	32	37	43	40	41	37	44	27	33	30	38	36

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## SECTION - C

6. (a) Calculate product moment correlation coefficient. 10

X	120	125	127	130	134	144
Y	42	47	48	46	50	49

- (b) (i) Write a short note on Scatter Diagram. 5  
(ii) Calculate spearman's rank correlation coefficient. 5

X	49	55	57	60	74
Y	17	15	24	18	31

- (c) Define Bivariate Normal Distribution. State and explain the properties of Bivariate normal distribution. 10  
(d) Find whether A and B are independent if  $(AB) = 256$ ,  $(\alpha B) = 768$ ,  $(A\beta) = 48$ ,  $(\alpha\beta) = 144$ . Also find coefficient of Association. 10

7. (a) Find the regression equation of Y on X and X on Y. 10

X	2	4	8	10	6
Y	1	2	0	4	5

- (b) (i) What is regression ? Why are there in general, two regression lines ? When do they coincide ? 5  
(ii) Calculate multiple correlation coefficient  $r_{1.23}$  if  $r_{12} = 0.98$ ,  $r_{13} = 0.44$ ,  $r_{23} = 0.54$  5
- (c) Define multinomial distribution and solve the following problem. 10  
A community consists of 50 percent Hindus, 30 percent Muslims and 20 percent Sikhs. If a sample of six individuals is selected at random, what is the probability that two are Hindus three are Muslims and one is a Sikh ?
- (d) Find the frequencies of the positive classes including the whole number of observations N if. 10  
 $(ABC) = 75$ ,  $(\alpha BC) = 98$   
 $(AB\gamma) = 310$ ,  $(\alpha B\gamma) = 702$   
 $(A\beta C) = 106$ ,  $(\alpha\beta C) = 74$   
 $(A\beta\gamma) = 489$ ,  $(\alpha\beta\gamma) = 8415$

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