2007

MATHEMATICS - II (Optional)

100057

Standard : Degree **Total Marks : 200**

Nature : Conventional **Duration : 3 Hours**

Note :

(i) *Answers must be written in* ***English*** *only.*

(ii) *Question* **No. 1** *is* ***Compulsory.*** *Of the remaining questions, attempt* ***any four*** *selecting one question from* ***each section.***

(iii) *Figures to the* ***RIGHT*** *indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference Book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing/presentation.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

Marks

1. ***Answer any four of the following :***

(a) Prove that if G is a finite group and H is a subgroup of G then O(H) is a divisor of O(G). Is the converse true ? Justify your answer. 10

(b) Prove that a necessary condition that a function $w=f(z)=u(x, y)+iv(x, y)$ is analytic at a point $z=x+iy$ of its domain D is that at point (x, y) the first order partial derivatives of u and v with respect to x and y exist and satisfy the Cauchy–Riemann equations. 10

$Ux=Vy$ and $Uy=-Vx$.

(c) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$. 10

P.T.O.

Marks

(d) Draw all possible trees with 5–vertices. **10**

(e) State and prove Bolzano–Weierstrass theorem. **10**

SECTION - A

2. Answer the following sub-questions :

(a) If p is a prime number and $p^{\alpha} \mid O(G)$, then G has a subgroup of order p^{α}. **15**

(b) Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element of R. **15**

(c) Let F (a) be smallest subfield of k containing both F and a. Prove that the element $a \in k$ is algebraic over F if and only if F (a) is a finite extension of F. **10**

3. Answer the following sub-questions :

(a) Let $\varnothing$ be a homomorphism of G onto $\overline{G}$ with kernel k and let $\overline{N}$ be a normal subgroup of $\overline{G}$, $N = \{x \in G \mid \varnothing(x) \in \overline{N}\}$. Prove that $G/N \approx \overline{G}/\overline{N}$. **15**

(b) Let $f(x) = a_0 + a_1 x + \ldots\ldots\ldots + a_n x^n$ be a polynomial with integer coefficients. Suppose that for some prime number p, $p \nmid a_n$, $p \mid a_1$, $p \mid a_2$, ... $p \mid a_0$, $p^2 \nmid a_0$. Prove that $f(x)$ is irreducible polynomial over rationals. Is the converse true ? Justify your answer. **15**

(c) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then prove that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b2 = 0$. **10**

SECTION - B

4. Answer the following sub-questions :

(a) State and prove Intermediate Value Theorem. **15**

(b) Prove that every continuous function in a closed interval [a, b] is uniformly continuous in that interval. **15**

(c) Find the Laurent's series of the function $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ in the annulus $2 < |z| < 3$. **10**

Marks

5. Answer the following sub-questions :

(a) If $f(x)$ is continuous in the closed interval [a, b] and differentiable at every point of the open interval (a, b) then prove that there exists at least one value θ, where $0 < \theta < 1$, such that $\frac{f(b)-f(a)}{b-a} = f'[a+\theta(b-a)]$. **15**

(b) Prove that a series $\sum_{n=1}^{\infty} g_n(x) = f(x)$ which converges uniformly in an interval can be integrated term by term in that interval. **15**

(c) Show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \int_0^{2\pi} \frac{d\theta}{a+b\sin\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$, by using contour integration. **10**

SECTION - C

6. Answer the following sub-questions :

(a) Solve the equation $(x+y)\,dx-(x-y)\,dy=0$. **15**

(b) Show by an example that the existence of directional derivative at a point need not imply the continuity of the function at that point. **10**

(c) Find the orthogonal trajectories on the cone $x^2+y^2=z^2\tan^2\alpha$ of its intersections with the family of planes parallel to $z=0$. **15**

7. Answer the following sub-questions :

(a) Test the equation $(\sin x\tan y+1)dx-\cos x\sec^2 y\,dy=0$ for exactness and solve it if it is exact. **15**

(b) Give an example of a function of two variables $f(x, y)$ for which $\frac{\partial^2 f}{\partial x\partial y} + \frac{\partial^2 f}{\partial y\partial x}$ at a point. Justify your answer. **10**

(c) Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$. **15**

P.T.O.

Marks

SECTION - D

8. Answer the following sub-questions :

(a) Derive Newton–Raphson formula to find the square root of a given positive number. Hence find $\sqrt{24}$. Start with $x_0 = 3$ and perform three iterations. **10**

(b) Solve the following system of linear equations by Gauss–Siedel iteration method, start with $(x, y, z) = (0, 0, 0)$ and do two iterations. **10**
$83x + 11y - 4z = 95,\ 7x + 52y + 13z = 104,\ 3x + 8y + 29z = 71.$

(c) Solve the following integration by Sympson's one third rule with $h = 0.2$ **10**
$\int_0^{1.2} \frac{dx}{1+x}$.

(d) Solve by graphical method the following linear programming problem. **10**
Minimize $Z = 3x_1 + 5x_2$
Subject to $-3x_1 + 4x_2 \leq 12$
$2x_1 - x_2 \geqslant -2$
$2x_1 + 3x_2 \geqslant 12$
$x_1 \leq 4,\ x_2 \geqslant 2$ and $x_1 \geqslant 0,\ x_2 \geqslant 0.$

9. Answer the following sub-questions :

(a) Find the real root of $x^3 - 2x - 5 = 0$ by regula falsi method, which lies between 2 and 3. Do three iterations. **10**

(b) Find the second degree polynomial passing through the points $f(-1) = 3, f(0) = 9, f(2) = 27$. Using Lagrange's interpolation formula. **10**

(c) Derive general quadrature formula. **10**

(d) Find the dual of the following linear programming problem. **10**
Minimize $Z = x_1 + 2x_2 + 3x_3 - x_4$
Subject to $x_1 + 2x_2 + 3x_3 = 15$
$2x_1 + x_2 + 5x_3 \leq 20$
$x_1 + 2x_2 + x_3 + x_4 = 10$
$x_1, x_2, x_3, x_4 \geqslant 0.$

-oOo-