AGU

2007

MATHEMATICS - I (Optional)

100053

Standard : Degree | *Total Marks : 200*

Nature : Conventional | *Duration : 3 Hours*

Note :

(i) *Answers must be written in* ***English.***

(ii) *Question* ***No. 1*** *is* ***Compulsory.*** *Of the remaining questions, attempt* ***any four*** *selecting one question from* ***each section.***

(iii) *Figures to the* ***RIGHT*** *indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he/she will be penalised.*

Marks

1. Answer **any four** of the following :

(a) Using Cayley-Hamilton theorem find the expression for the inverse of a non-singular matrix and hence find A^{-1} for $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$. **10**

(b) Discuss the convergence of the following integral : **10**

$$I = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

m and n are both positive.

(c) Suppose U is an open set in $\mathbb{R}^3$ and $F : U \rightarrow \mathbb{R}^3$ be a continuously differentiable vector field. **10**

Define : (i) curl F (ii) div F

Prove that curl (curl F) = grad (div F) − Δ^2F

P.T.O.

Marks

(d) A uniform rod OA of length 2a, free to turn about its end O, revolves with uniform angular velocity ω about the vertical OZ through O, and is inclined at a constant angle α to OZ, show that the value of α is either zero or $\cos^{-1}(3g/4a\omega^2)$. 10

(e) Find a, b, c if 10

$$\lim_{x \to 0} \frac{a\,x\,e^{x} - b\log(1+x)\; c\,x\,e^{-x}}{x^2 \sin x} = 2$$

SECTION - A

2. Answer the following sub-questions :

(a) If V is a vector space over an infinite field F, then prove that V cannot be written as set - theoretic union of a finite number of proper subspaces. 10

(b) If $B = \{e_1 = (1, 0),\ e_2 = (0, 1)\}$ is the standard basis of vector space $V = \mathbb{R}^2$ and if $T : V \to V$ is a linear transformation given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 - e_2$, also if $v = \alpha_1 e_1 + \alpha_2 e_2$ ($\alpha_1, \alpha_2 \in \mathbb{R}$) is a eigen vector of T, then show that $\pm\sqrt{2}$ are the only eigen values of T and find corresponding eigen-vectors of T. Further show that these eigen vectors form an ordered basis of V and find the matrix of T relative to this basis. ($\mathbb{R}$ is the set of all real numbers) 10

(c) Find an orthogonal matrix A whose first row is $u_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ 10

Is the above matrix A unique ?

(d) Find the symmetric matrix A, which corresponds to the following quadratic (polynomial) form. 10

$3x^2 + 4xy - y^2 + 8xz - 6yz + z^2$.

Determine whether the above symmetric matrix is positive definite.

Marks

3. Answer the following sub-questions :

(a) Find the condition on the scaler ξ so that the vectors (1, 1, 1) and $(1, \xi, \xi^2)$ form a basis of C^3 (C is the set of all complex numbers) **10**

(b) Find a homogeneous system whose solution set W is generated by $\{ (1, -2, 0, 3), (1, -1, -1, 4), (1, 0, -2, 5) \}$ **10**

(c) Let V be a vector space and $\omega \epsilon V$ be a vector such that inner product $\langle w, w \rangle \neq 0$. Let $W = \{ cw / c \epsilon \mathbb{R} \}$ be the span of w prove that V is the direct sum of W and its orthogonal complement $W^{\perp}$. **10**

(d) Reduce the following quadratic form into standard form and name the conic $11x^2 + 6xy + 19y^2 - 80$ **10**

SECTION - B

4. Answer the following sub-questions :

(a) If the function $f : [0, 4] \to \mathbb{R}$ is differentiable then prove that $(f(4)^2 - f(0)^2) = 8 f'(a) f(b)$ for same a and $b \in [0, 4]$. **15**

(b) If $f(x, y) = \frac{x}{y}$ if $|y| \geq |x|$ **15**

$= \frac{y}{x}$ if $|x| \geq |y|$

$= 0$ if $x = y = 0$

then (i) Find $f_x(0, 0)$, $f_{xy}(0, 0)$

(ii) Discuss the continuity of f at (0, 0)

(iii) Discuss the differentiability of f at (0, 0)

(c) Find the area and the centre of gravity of the plane semi-circular region $x^2 + y^2 \leq 4$, $x \geqslant 0$. **10**

5. Answer the following sub-questions :

(a) Prove that the number θ which occurs in Taylor's theorem with the Lagrange's form of remainder after n terms approaches the limit $\frac{1}{n+1}$ as h approaches zero provided that $f^{n+1}(x)$ is continuous and different from zero at $x = a$. **15**

(b) A rectangular box without top with volume 108 cubic units is to be constructed from a sheet of metal. Find the dimensions of the box if the least amount of material is to be used in its manufacture. **15**

(c) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. **10**

Marks

SECTION - C

6. (a) Sketch the graph of the function **10**

$f(x,y)=\sqrt{x^2+y^2+1}$

(b) Define norm of a vector in $\mathbb{R}^2$ and $\mathbb{R}^3$. Prove that $||u+v||^2+||u-v||^2=2||u||^2+2||v||^2$, where $u, v \in \mathbb{R}^2$ and interpret the result geometrically by translating it into a theorem about parallelogram. **15**

Let $u=(x, y, z)$, Describe that set

(i) $||u||=1$, (ii) $||u||<1$, (iii) $||u||>1$

(c) Derive the expression for cylindrical and spherical co-ordinates of a point in $\mathbb{R}^3$. Transform the Cartesian co-ordinates of a point $(4\sqrt{3}, 4, -4)$ into cylindrical co-ordinates. Also transform spherical co-ordinates of a point $\left(4, \frac{\pi}{6}, 3\right)$ into Cartesian co-ordinates. **15**

7. (a) What are the quadric surfaces ? Explain different forms of quadric surfaces. Explain them with illustrations. **10**

(b) Find the distance between the skew lines $L_1 : x=1+7t,\ y=3+t,\ z=5-3t$ and $L_2 : x=4-t,\ y=6,\ z=7+2t$. Show that the line $x=-1+t,\ y=3+2t,\ z=-t$ and the plane $2x-2y-2z+3=0$ are parallel and find the distance between them. **15**

(c) If $u=\log(ax^2+2hxy+by^2)$ then show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}+2=0$. **15**

SECTION - D

8. (a) A particle is attached to the middle point of an elastic string stretched between two points, A and B on a smooth horizontal table. If the particle be displaced through a small distance perpendicular to AB and then released, show that the motion is Simple Harmonic Motion. **15**

(b) What are the forces which are neglected in forming the equation of virtual work and why ? **15**

(c) Uniform solid circular cylinder rolls down on inclined rough plane. Discuss the motion. **10**

P.T.O.

Marks

9. (a) A particle describes an ellipse as a central orbit about the forces. Prove that the velocity at the end of the minor axis is geometric mean between the velocities at the ends of any diameter. 15

(b) A heavy uniform rod rests with one end against a smooth vertical wall and with a point at its length resting on a smooth peg. Find the position of equilibrium and show that it is unstable. 15

(c) Find moment of inertia of an elliptic disc $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. 10

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