2006

STATISTICS - I (Optional)

000054

Standard : Degree | *Total Marks : 200*

Nature : Conventional | *Duration : 3 Hours*

Note :

(i) *Answers must be written in* **English** *only.*

(ii) *Question* **No. 1** *is* **Compulsory.** *Of the remaining questions, attempt* **any four** *selecting one question from* **each section.**

(iii) *Figures to the* **RIGHT** *indicate marks of the respective question.*

(iv) *Through use of log table, Non-Programmable calculator is permitted, but any other Table/Code/ Reference book are not permitted.*

(v) *Make suitable assumptions, wherever be necessary and state the same.*

(vi) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vii) *Credit will be given for orderly, concise and effective writing.*

(viii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he will be penalised.*

1. Answer *any four* of the following :

(a) (I) Define the following with suitable examples : 5

(i) Discrete random variable (*r.v.*)

(ii) Probability Mass function (*p.m.f.*)

(iii) Distribution function (*d.f.*)

(II) In a quality control department of a rubber tube manufacturing factory 10 rubber tubes are randomly selected from each day's production for inspection. If not more than one of the 10 tubes is found to be defective, the production lot is approved. Otherwise it is rejected. Find the probability of the rejection of a day's production lot if the true proportion of defectives in the lot is 0.3. 5

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(b) Define the following with illustrations : **10**

(i) Simple and composite hypothesis.

(ii) Type-I and Type-II errors.

(iii) p-value.

(iv) Power of a test.

(v) Critical region.

(c) Let Y_1, Y_2 and Y_3 be the yield in kilograms of a crop, rainfall in mm and maximum temperature in °F of a place. A sample on Y_1, Y_2, Y_3 gave following results. **10**

$\overline{Y}_1 = 103.9214, \ \overline{Y}_2 = 93.9714, \ \overline{Y}_3 = 99.7357$

$\sigma_1 = 24.4492, \ \sigma_2 = 45.4930, \ \sigma_3 = 2.9315$

$r_{12} = 0.4104, \ r_{13} = -0.7357, \ r_{23} = -0.2326$

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(i) Obtain the equation of plane of regression of Y_1 on Y_2 and Y_3.

(ii) Estimate Y_1 when $Y_2 = 98$ mm and $Y_3 = 20$°F

(d) Explain stratified random sampling. What do you mean by allowcation of sample size ? Explain different types of allowcation and obtain the expression for sample size in each case. **10**

(e) Explain the following with illustrations : **10**

(i) Sign-test (ii) Run test, for single sample.

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SECTION - A

2. *Answer the following sub-questions* :

(a) (i) Suppose heights of soldiers follow normal distribution with mean 170 cm and variance 50 cm^2. In a regiment of 1000 soldiers how many would you expect to be over 180 cm tall. 5

(ii) State and prove weak law of large numbers. 5

(b) Let the joint *p.m.f.* of two discrete *r.v.s.*, X and Y be : 10

$$p(x,y)=\frac{n!}{x!\,y!(n-x-y)!}\,p_1^x\,p_2^y\,p_3^{n-x-y}$$

where x, y and n are non-negative integers with

$0 \le x \le n$; $0 \le y \le n$; $0 \le (x+y) \le n$; $0 < p_1, p_2, p_3 < 1$ with $p_1+p_2+p_3=1$

Find (i) Marginal *p.m.f.* of X and Y

(ii) Conditional *p.m.f.* of X given $Y=y$

(c) Two dice are thrown once. If X is the sum of the numbers showing up, prove that $P\{|X-7| \geq 3\} \leq 35/54$, and compare this value with exact probability. 10

(d) State and prove central limit theorem. Also state any two applications of it. 10

3. *Answer the following sub-questions :*

(a) (I) Define the following with illustrations. 5

(i) Sample space

(ii) Event

(iii) Complementary event

(iv) Mutually exclusive events

(v) Equally likely events

(II) Let $X_1, X_2, \ldots X_n$ be i.i.d variates with mean μ, and variance σ^2 as $n \to \infty$,

$$(X_1^2+X_2^2+\ldots\ldots+X_n^2)/n \xrightarrow{P} C$$

for some constant C, $(0 \le C \le \infty)$. Find C. 5

(b) The joint *p.m.f.* of X_1 and X_2 is 10

$P[X_1=x_1, X_2=x_2] = k(2x_1+5x_2)$

where (x_1, x_2) = (1, 1); (1, 2); (2, 1); (2, 2).

(i) Determine k

(ii) Obtain conditional mean and variance of X_2 given $X_1=2$

(iii) Are X_1 and X_2 independent ?

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(c) Define moment generating function *(m.g.f.)* and probability generating function *(p.g.f.)* of a random variable. Also state their properties. **10**

(d) Define finite Markov chain. Also explain the following : **10**

(i) Persistent state

(ii) Transient state

(iii) Periodic state

(iv) null state

(v) ergodic state

SECTION-B

4. Answer the following sub-questions :

(a) State and prove Neyman's Factorization theorem in case of discrete random variables. **10**

(b) (i) Let X_1, X_2 be i.i.d. Pois(θ) variates. Show that the statistic $T = X_1 + 3X_2$ is not sufficient for θ. **5**

(ii) For a random sample of size n from $N(\mu, \sigma^2)$ population find the MLE for μ when σ^2 is known **5**

(c) Define Uniformly Most Powerfull Test (UMPT). **10**

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Given $f(x, \theta) = 1 + \theta^2 \left[x - \frac{1}{2}\right]; \ 0 \le x \le 1, 0 \le \theta \le \sqrt{2}$

Find a best critical region of size $\alpha = 0.1$ for testing $H_0 : \theta = 0$ against $H_1 : \theta \ne 0$ based on a single value of X. Is this a UMPT ?

(d) Explain the following : **10**

(i) Kolmogoroff - Smirnov's test for goodness of fit.

(ii) Wilcoxon signed — ranks test.

5. Answer the following sub-questions :

(a) A Uniformly Minimum Variance Unbiased Estimater (UMVUE) is unique in the sense that if T_1 and T_2 are UMVUEs for a parameter θ, then $T_1 = T_2$. Prove the above statement. **10**

(b) (i) Let $X_1, X_2, \ldots\ldots, X_n$ are random observations on a Bernoulli variate X taking the value one with probability θ and value zero with probability $(1 - \theta)$. Show that $T\ (T-1)/n\ (n-1)$ is an unbiased of θ^2, where $T = X_1 + X_2 + \ldots\ldots X_n$. **5**

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(ii) Prove that if $T = T(X)$ is a sufficient statistic for a parameter θ and a unique MLE $\hat{\theta}$ of θ exists, then $\hat{\theta}$ is a function of T. More over if any MLE exists, an MLE $\hat{\theta}$ can be found which is a function of T. 5

(c) A drug was administered to 10 patients and the increments in their blood-pressure were recorded to be 6, 3, -2, 4, -3, 4, 6, 0, 0, 2. Is it reasonable to believe that the drug has no effect on change of blood-pressure ? 10

[Take $\alpha = 0.05$, $p[\ t > 2.26\ ] = 0.025$]

(d) Let X follow $N(\theta, \sigma^2)$ where σ^2 is known. For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ construct the SPRT and obtain its ASN. 10

SECTION - C

6. Answer the following sub-questions :

(a) What is correlation ? Explain different types of correlation and various measures of correlation. 10

(b) (i) Find correlation coefficient between X and Y, given that $n = 25$, $\sum x = 75, \sum y = 100, \sum x^2 = 250, \sum y^2 = 500, \sum xy = 325.$ 5

(ii) In a trivariate distribution : 5

$$\sigma_1^2 = 4, \sigma_2^2 = \sigma_3^2 = 9, r_{12} = 0.7, r_{13} = r_{23} = 0.5$$

Find $R_{1.23}$, $r_{13.2}$ and $\sigma_{1.23}$

(c) Define multinomial distribution. Obtain its : 10

(i) MGF

(ii) Marginal distribution

(iii) Conditional distribution

(d) Test whether the following data are consistent. "45% students have passed in mathematics and statistics, 35% students passed in mathematics but failed in statistics, 25% students have passed in statistics but failed in mathematics." 10

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7. *Answer the following sub-questions :*

(a) Given $x-4y=5$ and $x-16y=-64$ are the regression lines, find — 10

(i) regression coefficient of x on y,

(ii) regression coefficient of y on x,

(iii) r_{xy}

(iv) $\bar{x}, \bar{y}$

(v) σ_y if $\sigma_x = 8$

(b) (i) Explain the principle of least square for obtaining regression line. — 5

(ii) If all the total correlation coefficients in a set of three variables equal to $\rho(\rho \neq 1)$ then show that — 5

(1) $R^2_{1.23} = \frac{2\rho^2}{1+\rho}$

(2) $r_{12.3} = \frac{\rho}{1+\rho}$

(c) If (X, Y) follows BVN (3, 1, 16, 25, 315) — 10

Find (i) $P[3 < Y < 8 / x = 7]$

(ii) $P[-3 < X < 3 / y = -4]$

(d) Can vaccination be regarded as a preventive measure for small-pox from the data given below ? — 10

Of 1482 persons in a locality exposed to small-pox 368 in all were attacked.

Of 1482 persons, 343 had been vaccinated and of these only 35 were attacked.

SECTION - D

8. *Answer the following sub-questions :*

(a) (I) Explain the following with illustrations. — 5

(i) Simple random sampling with and without replacement.

(ii) Stratified random sampling.

(II) Explain Ratio and regression methods of estimation. — 5

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(b) Explain systematic sampling with illustration. Also obtain the expression for variance of estimated mean. **10**

(c) Explain the principal steps in sample survey. **10**

(d) Write a note on the following : **10**

(i) NSSO

(ii) CSO

(iii) ISI

9. *Answer the following sub-questions :*

(a) (i) Obtain the optimum values of sample sizes for different strata in case of Neyman's optimum allocation. **5**

(ii) Explain the procedure of drawing a sample in case of cluster sampling. Also compare it with stratified random sampling. **5**

(b) Define Ratio estimator for estimating population total of a character y and derive an expression for the standard error of the estimator. **10**

(c) Describe briefly the 'Questionnaire' method of collecting the primary data. Also state the requirements of a good questionnaire. **10**

(d) Describe various methods of collection of official statistics and their limitations. Also state principal publications publishing official statistics. **10**

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