BXD

2006

MATHEMATICS - II (Optional)

000062

Standard : Degree **Total Marks : 200**

Nature : Conventional **Duration : 3 Hours**

Note :

(i) *Answers must be written in **English**.*

(ii) *Question **No. 1** is **Compulsory**. Of the remaining questions, attempt **any four** selecting one question from **each section**.*

(iii) *Figures to the **RIGHT** indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference Book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he will be penalised.*

Marks

1. ***Answer any four of the following : (10 Marks each)***

(a) If S_n denotes the permutation group on n symbols, then prove that S_n contains $\frac{n!}{2}$ even permutations. **10**

(b) Using Cauchy's residue theorem, prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\,dx = \frac{\pi}{\sqrt{2}}$. **10**

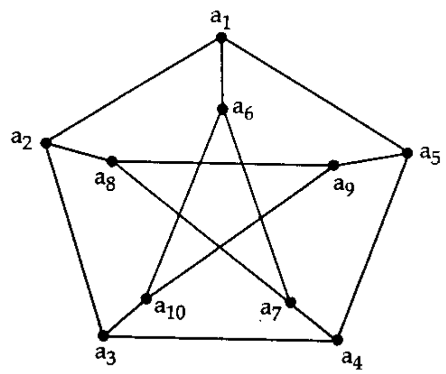
P.T.O.

Marks

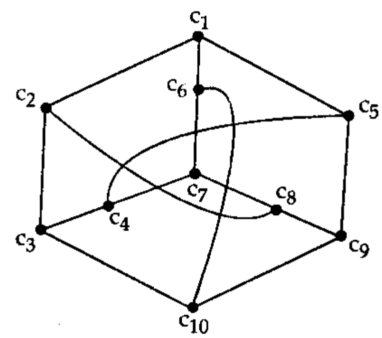
(c) Find the surface passing through the two lines $z = x = 0$, $z - 1 = x - y = 0$ satisfying $r - 4s + 4t = 0$. **10**

(d) (i) Define spanning tree of a connected graph and show that every connected graph has at least one spanning tree. **5**

(ii) Are the following two graphs isomorphic ? Justify your answer. **5**



graph - 1 (G_1)



graph - 2 (G_2)

(e) Show that a set in a metric space is open if and only if its complement is closed. **10**

SECTION - A

2. *Answer the following sub-questions :*

(a) Given H, K subgroups of a group G, show **15**

(i) HI K is a subgroup of G

(ii) HK + {hk : h∊H, k∊K} is a subgroup of G if and only if HK = KH.

(iii) HYK need not be a subgroup of G by giving a counter example.

(b) Define Euclidean ring. **15**

Show that the ring R of Gaussian ring $\{x + iy : x, y \in I\}$ is an Euclidean ring.

(c) Let F be a finite field with q elements and FCK where K is also a finite field then K has q^n elements where $n = [K : F]$. **10**

Marks

3. *Answer the following sub-questions :*

(a) Let H be a normal subgroup of G and K be a normal subgroup of G containing H. Show that $G/K \cong (G/H)/(K/H)$. **15**

(b) Define principal ideal domain.
Show that the polynomial ring $F[x]$ over field F is a principal ideal domain. **15**

(c) If L is a finite extension field of K and K is a finite extension field of F, then show that L is a finite extension of F and $[L : F] = [L : K] [K : F]$. **10**

SECTION - B

4. *Answer the following sub-questions :*

(a) Define complete metric space. Show that if X is a complete metric space then $Y \subset X$ is complete if and only if Y is closed in X. **15**

(b) Define uniform convergence of sequence of functions on a metric space E. **15**
If $\{f_n\}$ is a sequence of continuous functions converging uniformly to a function f on E, then show that f is continuous on E.

(c) Prove that the Cauchy Riemann conditions are necessary but not sufficient for a function f to be analytic. **10**

5. *Answer the following sub-questions :*

(a) Define uniformly continuous function from a metric space X to metric space Y. **15**
Show that if f is continuous real valued function on a compact metric space, then f is uniformly continuous.

(b) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point $x_0 \in [a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then show that $\{f_n\}$ converges uniformly on $[a, b]$ to a function f, and $f'(x) = \lim_{n \to \infty} f_n'(x), \ (a \le x \le b)$. **15**

(c) Show that every power series with positive radius of convergence defines an analytic function inside the circle of convergence. **10**

Marks

9. (a) Find the real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position, correct to two decimal places. 10

(b) The population of a town in decennial census is given below. Estimate the population for the year 1955 by using Newton's interpolation. 10

Year	1921	1931	1941	1951	1961
Population (In thousand)	46	66	81	93	101

(c) (i) If $f(x)$ is of the form $a + bx + cx^2$ find a formula for $\int_0^1 f(x)\,dx$ in terms of $f(0)$, $f(1)$ and $f(2)$. 5

(ii) Find the value of $log_e 7$ by using Simpson's rule. 5

(d) (i) Write the following L.P.P. in terms of it's dual : 5

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

(ii) A Company sells two different products *A* and *B*. The Company makes a profit of Rs. 30 and Rs. 40 per unit on the product *A* and *B* respectively. The two products are produced in a common production process and are sold in two different markets. The production process has a capacity of 35,000 man hours. It takes two hours to produce one unit of *A* and 3 hours to produce one unit of *B*. The market has been surveyed and Company feels that the maximum number of units of *A* that can be sold is 12000 units and maximum of *B* 8000 units. Assuming that the products can be sold, in any circumstances, formulate the L.P.P. 5

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Marks

SECTION - C

6. (a) (i) Find the differential equation of the family of circles of fixed radius with centres on [illegible]-axis. 5

(ii) Solve : $(x-y)\, dy=(x+y+1)\, dx$. 5

(iii) Integrate $(x^3e^x-my^2)\, dx+mxy\, dy=0$. 5

(b) (i) Find the directional derivative of the function $xy^2+yz^2+zx^2$ along the tangent to the curve $x=t,\ y=t^2,\ z=t^3$ at (1, 1, 1). 5

(ii) Let $f:R^2\longrightarrow R$ be a function defined as $f(x,y)=\dfrac{xy(x^2-y^2)}{x^2+y^2}$ 5

if $(x,y)\neq(0,0)$ and $f(0,0)=0$, show that $f_{xy}(0,0)\neq f_{yx}(0,0)$.

(c) (i) Solve : $\dfrac{dx}{x^2-yx}=\dfrac{dy}{y^2-zx}=\dfrac{dz}{z^2-xy}$ 5

(ii) Form a partial differential equation by eliminating the function f from 5

$z=y^2+2f\left(\dfrac{1}{x}+\log y\right)$.

(iii) Find the orthogonal trajectories of the cardiods $r=a(1-\cos\theta)$ where 'a' is the parameter. 5

7. (a) (i) Find the general and singular solution of the equation. 5

$$y=xp+\frac{ap}{\sqrt{1+p^2}} \text{ where } p=\frac{dy}{dx}.$$

(ii) Solve : $x^2\left(\dfrac{dy}{dx}\right)^2+xy\dfrac{dy}{dx}-6y^2=0$ 5

(iii) Solve : $xy-\dfrac{dy}{dx}=y^3e^{-x^2}$ 5

P.T.O.

Marks

(b) (i) If $u=x^2 tan^{-1}\left(\frac{y}{x}\right)-y^2 tan^{-1}\left(\frac{x}{y}\right)$ then show that $\frac{\partial^2 u}{\partial x \partial y}=\frac{x^2-y^2}{x^2+y^2}$. 5

(ii) State Lagrange's mean Value theorem and explain it's geometrical meaning. 5

(c) (i) Find the complete solution of the partial differential equation $p=(qy+z)^2$ by Charpit's method. 7

(ii) Find the characterstics of the partial differential equation $pq=z$ and determine the integral surface, which passes through the parabola $x=0, y^2=z$. 8

SECTION - D

8. (a) (i) By Newton-Raphson method find a root of the equation $x^5+5x+1=0$. 5

(ii) Find the root of equation $x^2-49=0$ by the method of bisection. 5

(b) Find the solution, to two decimals of the system of equations by Gauss-Seidel iteration method. 10

$$83x+11y-4z=95$$
$$7x+52y+13z=104$$
$$3x+8y-29z=71$$

(c) (i) Evaluate $\int_0^1 \frac{dx}{1+x}$ with $h=1/6$ by Simpson's one-third rule and compare the result with actual value. Given that $\log_e 2=0.6930$. 5

(ii) Evaluate $\int_0^1 cosx\, dx$, by using trapezoidal rule, Take $h=0.2$. 5

(d) Maximize $Z=5x_1+3x_2$ by graphical method, subject to 10

$3x_1+5x_2 \leq 15$
$6x_1+2x_2 \leq 24$
$x_1, x_2 \geqslant 0$

P.T.O.