BWD

2006

MATHEMATICS - I (Optional)

000063

Standard : Degree | *Total Marks : 200*

Nature : Conventional | *Duration : 3 Hours*

Note :

(i) *Answers must be written in* **English.**

(ii) *Question* **No. 1** *is* **Compulsory.** *Of the remaining questions, attempt* **any four** *selecting one question from* **each section.**

(iii) *Figures to the* **RIGHT** *indicate marks of the respective question.*

(iv) *Use of log table, Non-Programmable calculator is permitted, but any other Table/Code/Reference book are not permitted.*

(v) *Number of optional questions upto the prescribed number in the order in which they have been solved will only be assessed. Excess answers will not be assessed.*

(vi) *Credit will be given for orderly, concise and effective writing.*

(vii) *Candidate should not write roll number, any name (including their own), signature, address or any indication of their identity anywhere inside the answer book otherwise he will be penalised.*

Marks

1. Answer ***any four*** of the following :

(a) (i) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. 5

(ii) Find the eigenvectors associated with the largest eigenvalue of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$. 5

(b) (i) Examine for convergence the integral $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$. 5

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Marks

(ii) Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$. 5

(c) Show that $\nabla o (\bar{f} \times \bar{g}) = (\nabla \times \bar{f}) o \bar{g} - (\nabla \times \bar{g}) o \bar{f}$. 10

(d) Obtain the Lagrangian and equations of motion for the double pendulum of lengths l_1 and l_2 with corresponding masses m_1 and m_2. 10

(e) (i) Show that the function $f(x) = |x-1|$ is continuous but not differentiable at $x = 1$. 5

(ii) Verify Rolle's theorem for the function $f(x) = \sin 2x$ in the interval $\left[0, \frac{\pi}{2}\right]$. 5

SECTION - A

2. Answer the following subquestions :

(a) (i) Show that the vectors (1, 2, 0), (2, 1, 2), (3, 1, 1) form a basis of IR^3 and express the vector (2, 3, 3) as a linear combination of them. 5

(ii) If $\bar{u}, \bar{v}, \bar{w}$ are linearly independent vectors, show that the vectors $\bar{u}+\bar{v}, \bar{u}-\bar{v}$ and $\bar{u}-2\bar{v}+\bar{w}$ are also linearly independent. 5

(b) (i) When two matrices are said to be equivalent ? Prove that this relation is reflexive, symmetric and transitive. 5

(ii) Reduce the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$ to its normal form. 5

(c) (i) Prove that any two vectors corresponding to two distinct eigenvalues of a Hermitian matrix are orthogonal. 5

(ii) Show that the product of two unitary matrices of the same order is unitary. 5

(d) Reduce the matrix $\begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$ to its diagonal form and interpret the result in terms of quadratic form. 10

Marks

3. (a) (i) Show that $W=\{(x, y, z); x+y+z=0\}$ is a subspace of $\mathbb{R}^3$. State the dimension of W. 5

(ii) By stating the result used by you, extend the set $B=\{(2, 1, 1), (1, 3, 0)\}$ to a basis of $\mathbb{R}^3$. 5

(b) (i) Find a basis of the column null space of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$. 5

(ii) Reduce the matrix $\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \\ -4 & -3 & 5 & -4 \end{bmatrix}$ to row echelon form. 5

(c) (i) Show that the modulus of each characteristic root of a unitary matrix is unity. 5

(ii) Show that the matrix $\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal. 5

(d) Find the rank, index and signature of the form $2x_1^2+2x_2^2+3x_3^2-4x_2x_3-4x_1x_3+2x_1x_2$. State whether it is positive definite form. 10

SECTION - B

4. (a) (i) A function $f(x)$ is defined as $f(x)=\dfrac{\log(1+x)-\log(1-x)}{x}$; for $x \neq 0$. Define $f(0)$ so that $f(x)$ will be continuous at $x=0$. 5

(ii) Find the point 'ξ' of Lagrange's mean value theorem applied to the function $f(x)=ax^2+bx+c$ in the interval $[\alpha, \beta]$. 5

(iii) Find the asymptotes of the curve given by the equation $y=\dfrac{x^2}{\sqrt{x^2-1}}$. 5

(b) (i) Show that the function 5

$$u=\frac{2xy}{x^2+y^2}\text{; when } x^2+y^2 \neq 0$$

$$= 0 \quad \text{; when } x^2+y^2=0$$

is continuous with respect to x and y separately, but is not continuous at (0, 0) with respect to these variables together.

Marks

9. Answer the following subquestions :

(a) A point executes simple harmonic motion such that in two of its positions the velocities are u and v and the corresponding accelerations are α, β. Show that the distance between the positions is $\frac{v^2 - u^2}{\alpha + \beta}$ and the amplitude of the motion is $[(v^2 - u^2)(\alpha^2 v^2 - \beta^2 u^2)]^{1/2}/(\alpha^2 - \beta^2)$. **15**

(b) A square of side $2a$ is placed with its plane vertical between two smooth pegs which are in the same horizontal line at a distance c apart. Show that it will be in equilibrium when the inclination of one of its edges to the horizon is either $\frac{\pi}{4}$ or $\frac{1}{2}\sin^{-1}\left(\frac{a^2 - c^2}{c^2}\right)$ using the principle of virtual work. **15**

(c) Show that the radial and transverse components of acceleration of a particle moving in a plane are given by $\ddot{r} - r\dot{\theta}^2$ and $\frac{1}{r}\frac{d}{dt}(r^2\,\dot{\theta})$ respectively. **10**

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Marks

(ii) If $x+y+z=4$, $y+z=uv$, $z=uvw$ then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)}=u^2v$. **5**

(iii) Test for maximum and minimum the function $f(x, y)=x^2+xy+y^2-2x-y$. **5**

(c) (i) Evaluate the double integral $\iint_R \frac{1}{x^4+y^2}\,dxdy$; where R is the region $y \geq x^2$, $x \geq 1$. **5**

(ii) By using triple integral find the volume of the region bounded by the plane $x+2y+3z=6$ with the coordinate planes. **5**

5. (a) (i) Expand by using Taylor's theorem, the function $f(x)=x^4-3x+2$ in powers of $x-1$. **5**

(ii) If $f''(x)$ is continuous at $x=0$, then evaluate $\lim_{x\to 0} \frac{2f(x)-3f(2x)+f(4x)}{x^2}$ in terms of $f''(0)$. **5**

(iii) Examine the function $f(x)=2x^3+3x^2-12x+5$ for its extreme values. **5**

(b) (i) If $u=x+\frac{x-y}{y-z}$ then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=1$. **5**

(ii) Find the extreme values of the functions $z=6-4x-3y$ subject to the constraint $x^2+y^2=1$. **5**

(iii) Show that the function $f(x, y)=|x|+|y|$ is continuous but not differentiable at (0, 0). **5**

(c) (i) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{1}{\sqrt{a^2-x^2-y^2-z^2}}\,dz\,dy\,dx$ **5**

(ii) Find the centre of gravity of the area enclosed by the curves $y^2=ax$ and $x^2=ay$. **5**

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Marks

SECTION - C

6. Answer the following subquestions :

(a) Prove that the general equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a conic section. **10**

(b) Find the equation of a cone whose vertex is (α, β, v) and whose guiding curve is a conic in xy-plane. **15**

(c) Define solenoidal and irrotational vector fields. Show that $r^n \bar{r}$ is irrotational for every integers. Find n if $r^n \bar{r}$ is solenoidal. **15**

7. Answer the following subquestions :

(a) Show that a necessary and sufficient condition that the homogeneous equation $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ may represent a pair of planes is **10**

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(b) Find the equation of sphere passing through (1, 0, 0), (0, 1, 0), (0, 0, 1) with radius as small as possible. **15**

(c) Prove that $\bar{b} \circ \nabla (\bar{a} \circ \nabla \frac{1}{r}) = \frac{3(\bar{a} \circ \bar{r})(\bar{b} \circ \bar{r})}{r^5} - \frac{\bar{a} \circ \bar{b}}{r^3}$. **15**

SECTION - D

8. Answer the following subquestions :

(a) State Kepler's three laws of planetary motion. Show that in a central force field, the areal velocity is constant and hence deduce Kepler's second law. **15**

(b) A uniform chain of length $2l$ has its ends fixed at two points on the same level. If the sag at the middle is h, then prove that the span is $\frac{l^2-h^2}{h} \log \frac{l+h}{l-h}$. **15**

(c) Derive Hamiltonian equations of motion from Lagrange's equation of motion. **10**

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